The Stabilization Value of Groundwater and Conjunctive Water Management under Uncertainty*

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The importance of managing ground and surface water conjunctively increases with water scarcity and with inter and intratemporal fluctuations in precipitations. Both factors are becoming critical in many parts of the world; the former due to increased water demand associated with economic and population growth and the latter due to climate change.

A conjunctive ground and surface water system appears in a number of forms, which differ according to the water sources. Surface water may consist of stream flows emanating from aquifers, surface reservoirs or lakes, snowmelt, rainfall or any combination. It may be stable or stochastically fluctuate over time. Groundwater sources—aquifers—may be nonreplenishable or replenishable, deep or shallow, confined or unconfined. The two cases in which only surface water or only groundwater is used lie on both ends of the conjunctive spectrum; these extreme cases occur when one source is always cheaper than the other (scarcity cost included). Conjunctive systems, viewed in this larger context, characterize most irrigation systems worldwide. The term conjunctive signifies that the ground and surface water sources are two components of one system and should be managed as such.

Oscar Burt pioneered the analysis of conjunctive water systems more than 40 years ago. More recently, Tsur (1990, 1997); Tsur and Graham-Tomasi; Provencher and Burt; and Knapp and Olson extended the theory to account for stochastic, dynamic, and multiaquifer considerations. The underlying idea is simple. Surface water sources derived from rainfall and snowmelt typically fluctuate randomly from year to year and within a year. Groundwater stocks, on

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the other hand, are relatively stable because the slow subsurface flows tend to
smooth out intra and intertemporal fluctuations. Groundwater thus performs
the dual functions of increasing the mean and reducing the variability of total
water supply. The value of groundwater is usually attributed to its
mean-increasing role, while its role in reducing variability is ignored. Yet, the
latter role carries an economic value, which is designated as the stabilization
value (or buffer values in the dynamic context) of groundwater, which could be
substantial.

Why should we be interested in the stabilization value as a distinct concept?
Suppose that a groundwater development project can be implemented at some
cost and the decision whether or not to undertake the project is based on a
cost–benefit criterion. Clearly, determining the benefit generated by the
groundwater project assuming that surface water is stable at the mean, while
easier to obtain, ignores the stochastic fluctuations of surface water and the
ensuing stabilization role of groundwater. If the economic value associated with
this role—the stabilization value—is nonnegligible compared to the value of the
resource due to increasing the mean water supply, this simpler approach leads
to a serious underestimation of the groundwater benefit and bias assessment of
the groundwater projects. Empirical studies (Tsur 1990, 1997) reveal substantial
stabilization value of groundwater.

We discuss implications of the stabilizing role of ground water for conjunctive
ground and surface water policies. Applying the analysis to Coimbatore Water
District in Tamil Nadu, India reveals a substantial stabilization value. We
conclude with some remarks regarding the important role of conjunctive water
management policies in a world of increasing food demands and declining
irrigation water supplies.

The Stabilization Value of Groundwater Revisited

Suppose that crop production requires only water and let $f(x)$ be the
per-hectare yield-water response function, with $x$ representing water input (the
empirical application employs an extended version with multiples outputs and
inputs). The water response function $f(x)$ is assumed increasing and strictly
concave over the appropriate range of water input, reflecting the diminishing
marginal productivity property. Let $R(x) = pf(x)$ represent revenue per hectare
when output price is $p$ (assumed exogenous to farmers). Following the
properties of the yield-water response function, $f(x)$, the revenue function, $R(x)$,
is increasing and strictly concave in water input.

Let $S$ represent available surface water supply (e.g., annual rainfall), assumed
to fluctuate randomly according to some probability distribution function $F(S)$.
(We ignore intraseasonal variations of surface water and consider only
variations in total supply of surface water during a year.) When surface water
(rainfall, stream flows emanating from snow melt) is the only source of
irrigation water, the revenue (which is also profit in this case) $R(S)$ also
fluctuates randomly around its mean $R_m = E\{R(S)\}$, where $E$ represents
expectation with respect to the distribution of $S$. If rainfall could be stabilized at
the mean $S_m = E\{S\}$, the revenue would have changed to $R(S_m)$. Since $R(\cdot)$ is
strictly concave, we have (Jensen’s inequality)
Let $S_{ce}$ be the Certainty-Equivalent water input satisfying
\[
E\{R(S)\} = R(S_{ce}),
\]
i.e., $S_{ce}$ is the constant annual surface water that leaves farmers indifferent between receiving it with certainty (every year) and facing the uncertain rainfall $S$ with distribution $F(S)$. Since $R(\cdot)$ is increasing, (1) and (2) imply $S_{ce} < S_m$. (Growers are assumed to be risk neutral and the divergence between $S_{ce}$ and $S_m$ stems from the concavity of the revenue function.)

How much farmers are willing to pay in order to stabilize surface water supplies at the mean ($S_m$) rather than facing the (actual) uncertain supplies $S$? The answer is simply the difference in revenues between the two situations: $R(S_m) - E\{R(S)\}$. We call this the Stabilization Value (SV) and, using (2), express it as
\[
SV \equiv R(S_m) - E\{R(S)\} = R(S_m) - R(S_{ce}).
\]

In figure 1, $SV$ is given by the area $HCS_mS_{ce}$.

To gain insight on how $SV$ depends on the distribution of rainfall, we take a second order Taylor expansion of $R(S)$ around $R(S_m)$ and approximate $E\{R(S)\}$ by
\[
E\{R(S)\} \approx R(S_m) + 0.5 R''(S_m) \sigma^2
\]
where $\sigma^2 = E\{(S - S_m)^2\}$ is the variance of $S$. By combining (3) and the above equation, we obtain
\[
SV \approx -0.5 R''(S_m) \sigma^2.
\]

We see from (4) that $SV$ increases with $\sigma^2$ (the variance of rainfall) and with $- R(S_m) = $ the steepness of the marginal revenue function (the derived demand for water) evaluated at $S_m$ (the average surface water supply). Thus, $SV$ increases with rainfall variability ($\sigma^2$) and with the steepness of the derived demand for water at the mean rainfall $S_m$. Typically, the derived demand for water is convex (i.e., the third derivative of $R(\cdot)$ is positive—see the examples in Tsur et al. and the application below). In such cases, the magnitude of $R''(S_m)$ decreases with $S_m$, so that $SV$ increases with the variance ($\sigma^2$) and decreases with the mean ($S_m$) of surface water supplies. We note that the purpose of approximation (4) is to shed light on how $SV$ depends on the distribution of surface water (particularly its mean and variance) but not for evaluation.

Suppose now that water from a groundwater source becomes available for irrigation at a constant unit cost $z$ ($\text{S} \text{ m}^{-3}$) and assume further that the rainfall distribution and $z$ are such that some groundwater will be demanded also during rainy years, i.e., $R'(S_{\text{max}}) > z$, where $S_{\text{max}}$ is maximal surface water supply (or the upper support of the distribution of $S$). We consider the situation
Rainfall is distributed between $S_{\text{min}}$ and $S_{\text{max}}$ with mean $S_m = E\{S\}$. The derived demand for irrigation water is the value of marginal water productivity $R'(x)$. Areas underneath the derived demand curve represent revenues. $z$ is the unit cost of groundwater. The total revenue is the area $AEx,O$ and the average groundwater cost is $E\{z(x(z) - S_m)\} = \text{area } \text{GE}(z)S_m$. The average profit is the area $\text{AEGS}_mO$. The total value of groundwater is the area $\text{HEGS}_mS_{\text{ce}}$, of which $SV = \text{area } \text{HCS}_mS_{\text{ce}}$ and $AV = \text{area } \text{CEG}$.

in which groundwater pumping decisions are made after the rainfall realization is observed. (Tsur [1990] analyzed the stabilization value under this assumption. Tsur and Graham-Tomasi called it “ex post” and considered also the case in which groundwater pumping decisions are made before the realization of $S$ is observed, which they refer to as the “ex ante.”) In this case, profit-seeking farmers demand the constant (stabilized) water input $x(z)$ satisfying $R'(x(z)) = z$ by augmenting the rainfall realization $S$ with the amount of groundwater $g = x(z) - S$ (figure 1).

The introduction of groundwater has, in effect, led to a stabilized water supply. We can imagine it occurring in two steps. First, surface water is stabilized at the mean $S_m$ and second, the surface water supply is augmented by the amount $g_m = x(z) - S_m$ of groundwater. The economic value associated with the first (stabilizing) step is the Stabilization Value of groundwater and is given by $SV$ of equation (3). The economic value associated with the second step is the value due to the increase in the mean water supply from $S_m$ to $x(z)$ and is denoted the Augmentation Value ($AV$) of groundwater:

\[
AV = [R(x(z)) - R(S_m)] - z[x(z) - S_m]
\]
The total value of groundwater is the sum \( SV + AV \). Notice that the price of groundwater \( z \) affects \( AV \) but not \( SV \).

In figure 1, the total revenue is the area \( AEx(z)O \) and the average groundwater cost is \( E\{z \cdot (x(z) - S)\} = z \cdot (x(z) - Sm) = \text{area } GEx(z)Sm \). The average profit is the area \( AEgs_mO \). The total value of groundwater is the area \( HEGs_mSce \), of which \( SV = \text{area } HCS_mSce \) and \( AV = \text{area } CEG \). In general, \( Sce \) decreases with the variability of rainfall, as can be seen from (2) and (4). Thus, a mean-preserving spread of the rainfall distribution, which increases variance while keeping the mean intact, will increase the \( SV \) of groundwater and vice versa.

In actual practice, multiple crops and inputs exist and the derived demand for irrigation water is modified accordingly. The \( SV \) of groundwater is calculated as explained above, using the aggregate derived demand for water. The framework can be extended to multiple water storage sources, such as multiple aquifers with varying pumping costs and surface reservoirs. Moreover, some of the water storage sources may also be stochastic. As long as they are not perfectly correlated with rainfall, they are capable of affecting the variability of water supply in a way that gives rise to a stabilization value.

Ignoring the rainfall variability, by assuming that rainfall is stable at the mean, amounts to assuming that \( SV = 0 \), which does away with the stabilizing role of groundwater. This may lead to severe undervaluation of groundwater and distort water management policies. Some policy implications are discussed in the next section, followed by an empirical assessment of the stabilization value of groundwater in the Coimbatore water district of Tamil Nadu, India.

**Conjunctive Management**

The stabilization value of groundwater affects water policies in a number of ways. First, it affects the optimal extraction decisions of a dynamic exploitation policy. To see this, consider the above conjunctive ground and surface water system over a long period of time. Denote the aquifer’s stock at time \( t \) by \( G_t \), which evolve in time according to

\[
dG_t/\text{dt} = M(G_t) - gt,
\]

where \( M(G) \) is natural recharge, assumed nonincreasing and concave in the aquifer’s stock \( G \), and time is taken to be continuous. The aquifer management problem entails finding the pumping policy \( \{gt, t \geq 0\} \) that maximizes the value

\[
V(G_0) = \max_{\{gt, t \geq 0\}} \int_0^\infty \left[ E_t\{R(S_t + gt)\} - z(G_t)gt \right] e^{-rt} \text{dt}
\]

subject to (6), given initial stock \( G_0 \) and feasibility constraints \( (gt \geq 0, G_t \geq 0) \), where \( r \) is the time rate of discount and pumping cost \( z(G) \) may depend on the aquifer’s stock. The expectation \( E_t \) is conditional on the realization of \( S_t \) being observed before the pumping decision \( g_t \) is made—a situation called “ex-post” by Tsur and Graham-Tomasi.
A detailed analysis of this model can be found in Tsur and Graham-Tomasi (1991). Here, we just note the condition

\begin{equation}
R'(S_t + g_t) = z(G_t) + V'(G_t),
\end{equation}

determining the optimal pumping decision at time \( t \), where \( V'(G_t) \) is the incremental value due to marginal chance in the groundwater stock \( G_t \) or the shadow price of groundwater (also known as user cost, in-situ value, and scarcity value). (The expectation is ignored because the realization \( S_t \) is assumed to be observed when \( g_t \) is chosen.)

If, however, surface water is assumed stabilized at the mean, the management problem changes to

\begin{equation}
V^m(G_0) = \max_{g_t} \int_0^\infty \left[ R(S_m + g_t) - z(G_t)g_t \right] e^{-rt} dt
\end{equation}

subject to (6) and feasibility (e.g., nonnegativity) constraints and the optimal pumping rule (8) becomes

\begin{equation}
R'(S_m + g_t) = z(G_t) + V''(G_t),
\end{equation}

where \( V''(G_t) \) is the groundwater shadow price when \( S_t \) is fixed at the mean \( S_m \).

Tsur and Graham-Tomasi showed (under certain conditions) that

\begin{equation}
V'(G_t) > V''(G_t).
\end{equation}

The shadow price of groundwater under stochastic surface water supplies is larger because of the added role of groundwater in stabilizing water supply and the ensuing economic value that goes with it. Thus, at any given groundwater stock \( G \), water users should pay more for the resource, hence pump less, under stochastic rainfall relative to a stabilized situation.

The Stabilization Value can also have a considerable effect on cost-benefit analyses of groundwater projects. Quite often the mere access to an aquifer requires investment in infrastructure, besides the operational costs associated with water pumping and conveyance. This cost should be compared to the benefit associated with developing the aquifer. Ignoring the stabilization value leads to underestimating the benefit associated with the development project. To see this, consider the case where extraction cost \( z \) is independent of the stock and the aquifer stock is at a steady state; i.e., average extraction just equals recharge: \( E\{g(S)\} = M(G) \). In this case, the shadow price of groundwater vanishes at a steady state (Tsur and Graham-Tomasi) and Condition (8) implies that \( S + g(S) = S_m + M(G) \). The value \( V(G) \) evaluated at a steady state is thus given by

\begin{align*}
V(G) &= \int_0^\infty E\{R(S + g(S)) - zg(S)e^{-rt} dt = \int_0^\infty R(S_m + M(G)) - zM(G)e^{-rt} dt \\
&= \frac{R(S_m + M(G)) - zM(G)}{r} = \frac{R(S_m)}{r} + \frac{R(S_m + M(G)) - R(S_m) - zM(G)}{r}.
\end{align*}
The present value without groundwater is simply

\[ V^S = \int_0^\infty E\{R(S)\}e^{-rt} \, dt = \frac{E\{R(S)\}}{r}. \]

Thus, the benefit associated with developing the aquifer is

\[ V(G) - V^S = \frac{R(S_m) - E\{R(S)\}}{r} + \frac{R(S_m + M(G)) - R(S_m) - zM(G)}{r}. \]

The first term on the right-hand side is the present value of the Stabilization Value of ground water. The second term is the present value of the Augmentation Value of groundwater. Assuming stable surface water supplies is equivalent to assuming that SV equals zero, hence biases downward the project’s benefit. The magnitude of the bias depends on the magnitude of SV.

Empirical Findings

In previous applications, the share of SV in the total value of groundwater was found to be substantial (Tsur 1990, 1997). We calculate the stabilization value of groundwater in the Coimbatore water district, located in Tamil Nadu, India. Irrigation water is derived from surface reservoirs, filled by the monsoon rains and distributed via a system of canals, and from local aquifers. About 50% of the irrigation water comes from surface sources, 45% from local wells (groundwater) and the remaining from other sources (Palanisami).

There are two monsoon periods—the Southwest monsoon from June to September; and the Northeast monsoon from October to December. The period between January and May is dry, but surface reservoirs allow distributing water throughout the year. The annual rainfall, thus, constitutes the available surface water supplies.

The data procedures used to calculate the stabilization value of groundwater are described in a longer version of this article (available upon request). Here, we only report the results (table 1). Economic values are measured in Rupees. We see that the stabilization value accounts for more than 25% of the total value of groundwater. Ignoring it, by assuming stable rainfall, would have led to underestimation of the value of groundwater by more than 25%.

Concluding Comments

Population growth and rising living standards lead to rapid increases in the demand for water. Since the average quantity of renewable fresh water available for use in any particular location is constant and water conveyance is an expensive operation, water has become a scarce resource in many parts of the world. Adding the prevalence of deteriorating water quality and the increased awareness for water-related environmental and social problems helps to understand why water resource management has become a critical policy challenge. In the region studied here, surface water has been relocated away from irrigation to meet the growing demands of other sectors in a way that
Table 1. Groundwater values (except for last row, all values are in Rs)

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R(S_m)$</td>
<td>Revenue at the mean</td>
<td>2,395,063,341</td>
</tr>
<tr>
<td>$E{R(S)}$</td>
<td>Mean revenue</td>
<td>2,342,143,999</td>
</tr>
<tr>
<td>$SV = R(S_m) − E{R(S)}$</td>
<td>Stabilization value</td>
<td>52,919,342</td>
</tr>
<tr>
<td>$R(\infty) =$</td>
<td>Revenue under unlimited water</td>
<td>2,552,096,009</td>
</tr>
<tr>
<td>$A = R(\infty) − R(S_m)$</td>
<td>Upper bound on Augmentation Value AV</td>
<td>157,032,667</td>
</tr>
<tr>
<td>$A + SV$</td>
<td>Upper bound on the total value of groundwater</td>
<td>209,952,009</td>
</tr>
<tr>
<td>$SV/(A + SV)$</td>
<td>Lower bound on share of SV in total value of groundwater</td>
<td>0.252</td>
</tr>
</tbody>
</table>

reduces the average quantity of surface water available for irrigation and at the same time increases its variability (the withdrawal for nonagricultural uses is larger during dry years than during wet years).

Worldwide irrigation water still consumes the bulk of the available renewable fresh water resources (over 70%). While irrigated agriculture is practiced on only about 18% of total cultivable land (267 million hectare in 1997, of which 75% are in developing countries), it produces over 40% of agricultural output (Gleick, World Bank, World Development Report 2000/2001). Irrigated area is expected to continue to expand to meet the food demand of a growing population (Food and Agriculture Organization 2000), but fresh water resources available for irrigation will at best remain fixed and most likely decline, stressing the need for improved efficiency of irrigation water.

There are ways to increase agricultural output without reliance on fresh water sources, such as improved crop variety (genetically or conventionally modified), appropriate water pricing and increased use of marginal sources (reclaimed, saline water). In this work, we focus on conjunctive management of ground and surface water. We note that crop production is affected not only by the quantity of water input but also by how this quantity is distributed within and between growing seasons. Owing to the random nature of precipitations, surface water supplies typically fluctuate randomly while groundwater sources are relatively more stable. The latter, thus, can be used to stabilize the supply of irrigation water, thereby increasing output over and above that expected due the increase in the average quantity of water input.

Applying the analysis to the Coimbatore district in Tamil Nadu, India, we found that the stabilization value of groundwater exceeds 25% of the total value of groundwater. Ignoring the stabilization value (by assuming that surface water is stable at the mean) leads to undervaluing groundwater by more than 25%. Put differently, under the prevailing rainfall variability, conjunctive management of ground and surface water in this region can increase water use efficiency by more than 25%. Groundwater resources are prevalent worldwide, yet often
mismanaged due to their common property feature. This is particularly true for the region studied here (Palanisami). Understanding the true value of groundwater is a necessary step toward a better management of this resource.

References


